Trellis Coding of Non-coherent Multiple Symbol Full Response M-ary CPFSK with Modulation Index $1/M$

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Abstract

This paper introduces a trellis coded modulation (TCM) scheme for non-coherent multiple full response M-ary CPFSK with modulation index $1/M$. A proper branch metric for the trellis decoder is obtained by employing a simple approximation of the modified Bessel function for large signal to noise ratio (SNR). Pairwise error probability of coded sequences is evaluated by applying a linear approximation to the Rician random variable. Examples are presented for trellis codings of non-coherent binary CPFSK by using the Ungerboeck's set partitioning method. Asymptotic upper bounds on bit error probability are evaluated for the given coded systems, and simulation results are also presented.

1 Introduction

Trellis Coded Modulation (TCM) [1] is developed to obtain coding gain without increasing bandwidth. Also multidimensional TCM [2], [3] and Multiple TCM [4] are developed for power and bandwidth efficiency. These schemes are originally developed for a coherent modulation system. TCM application to multiple symbol differential phase shift keying (MDPSK) can be found in [5]. The paper [6] considers the multiple full response continuous phase frequency shift keying with non-coherent detection. This paper introduces the equivalent normalized squared distance (ENSD). The ENSD of non-coherent system plays the same role as the normalized squared Euclidean distance of coherent system for evaluating the bit error probability.

We show that a combination of a trellis encoder and a non-coherent N-consecutive M-ary CPFSK can potentially yield a significant improvement in performance, even for small N, over the uncoded system. For the analysis we use a linear approximation for the Rician random variable to evaluate the pairwise error probability. We show that the pairwise error probability of coded sequences can be expresaed as a function of the sum of ENSDs. We introduce the equivalent squared free distance, $d^2_{e,free}$, (which represent the smallest distance between coded sequences leaving from the same trellis state at a given time and remerging again later on). $d^2_{e,free}$ is a designing tool for the trellis encoder with a non-coherent system as the Euclidean squared free distance is for a coherent system.

2 System Model for TCM

Figure 1 is a simplified block diagram of system under investigation. Input bits, b_m, occurring at a rate R_b, are passed through a trellis encoder with code rate r, producing an encoded bit stream c_m at a rate, $R_s = rR_b$. These encoded bits, c_m, are converted to a sequence, $u_m = (u_{m,0}, u_{m,1}, \ldots, u_{m,N-1})$, where $u_{m,i} \in \{0, 1, 2, \ldots, M-1\}$. The N-dimensional vector, u_m, is fed into an N-*consecutive continuous phase encoder'* (NCPE) [7], where the state of NCPE, is denoted by V_m. The output is mapped into an

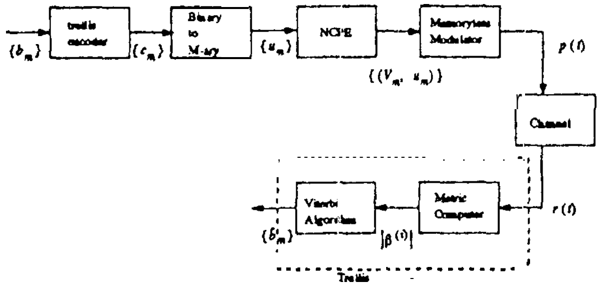

Figure 1: Block Diagram of the Trellis Coded Non-coherent N-consecutive M-ary CPFSK system

N-consecutive M-ary CPFSK waveform. This generation of CPFSK is explained in [7].

We assume that an arbitrary phase offset, θ_m, introduced by the channel during the reception of u_m is constant and uniformly distributed in [0, 2π). Furthermore the sequence of random variables are assumed to be independent (This assumption is valid if interleaving and deinterleaving is used after trellis encoder and before the Viterbi algorithm). At the receiver, the noise corrupted signal is non-coherently detected, and the resulting computed metrics, denoted by $|\beta^{(i)}|$, are then used for the branch values of the trellis. For coherent detection, a metric based on minimizing the squared Euclidean distance between received and transmitted waveforms, is optimum in the sense of a minimum probability of error sequence. For non-coherent detection, by the suitable modification, the appropate metric can be interpreted as an equivalent normalized squared distance (ENSD).

We denote a coded symbol sequence of length L corresponding to the output sequence of trellis encoder, $\mathbf{u} = (u_0, u_1, \ldots, u_{L-1})$, where the $m+1$st element of $\mathbf{u}$ is $u_m = (u_{m,0}, u_{m,1}, \ldots, u_{m,N-1})$. The state of NCPE, V_m, and the input vector, u_m, which specify a transmitting N-consecutive M-ary CPFSK waveform during the mth time interval $[mT_N, (m+1)T_N)$ where $T_N = NT$ and T is the duration of each coded symbol. The complex received baseband signal $\tilde{r}(t)$ can be represented aa $\tilde{r}(\tau + mT_N) = \tilde{s}(\tau + mT_N, u_m)e^{j\theta_m} + \tilde{n}(\tau)$ where

$$\tilde{s}(\tau + mT_N, u_m) \triangleq Ae^{j\Theta_N(\tau + mT_N, u_m)} \tag{1}$$

and

$$\Theta_N(\tau + mT_N, u_m) \triangleq \begin{cases} \frac{2\pi u_{m,0}}{M}\frac{\tau}{T}, & \text{if} \quad 0 \le \tau < T, \\ \frac{2\pi}{M}\sum_{k=0}^{i-1} u_{m,k} + \frac{2\pi}{M}\frac{u_{m,i}}{T}(\tau - iT), & \text{if} \quad iT \le \tau < (i+1)T, \\ \text{for } i = 1, 2, \cdots, N-1. \end{cases} \tag{2}$$

Observe that if we assume non-coherent detection, the phase information will not exist at the receiver. The Viterbi al-

. gorithm can be implemented with the same number of states as required for the trellis encoder.

The conditional probability of $\tilde{r}(t)$ over the interval $[0, (L-1)T_N)$, given an input sequence of length L, **u**, and carrier phase offsets, $\theta_0, \theta_1, ..., \theta_{L-1}$, can be expressed ss

$$P_r(\tilde{r}(t)|\mathbf{u}, \theta_0, \theta_1, \dots, \theta_{L-1}) \tag{3}$$
$$= C \prod_{m=0}^{L-1} \exp\{-\frac{1}{N_0}\int^{T_N} |\tilde{r}(\tau + mT_N)$$
$$\tilde{s}(\tau + mT_N, u_m)e^{j\theta_m}|^2 d\tau\}$$

which can be expressed in a simplified form

$$P_r(\tilde{r}(t)|\mathbf{u}, \theta_0, \theta_1, \dots, \theta_{L-1}) \tag{4}$$
$$= Ce^{-A^2 LT_N/N_0}$$
$$\exp\{-\frac{1}{N_O}\int_0^{L\ T_m} |\tilde{r}(t)|^2 dt\}$$
$$\prod_{m=0}^{L-1} \exp\{\frac{2A}{N_0}|\beta_m| \cos(\theta_m - \arg\beta)\}$$

where

$$\beta_m = \int_0^{TN} \tilde{r}(\tau + mT_N)e^{-j\Theta_N(\tau+mT_N, u_m)} d\tau. \tag{5}$$

Averaging (5) over $\theta_0, \theta_1, ..., \theta_{L-1}$, (assuming interleaving and deinterleaving is used) we get

$$P_r(\tilde{r}(t)|\mathbf{u}) = F\prod_{m=0}^{L-1} I_0(\frac{2A}{N_0}|\beta_m|), \tag{6}$$

where F is a constant which is independent of input sequence u. An exact evaluation of (6) in a closed form is difficult if not impossible. To get decision statistic which can be used to implement the Viterbi algorithm, we have employed a simple approximation of the modified Bessel function $I_0(z)$ for large SNR as follows:

$$\ln I_0(x) \approx |x|. \tag{7}$$

Therefore the decision variable behaves as

$$L(\tilde{r}(t)|\mathbf{u}) = \sum_{m=0}^{L-1} |\beta_m|. \tag{8}$$

Thus, the appropriate decision rule for the coded sequence **u** is the following:

$$\text{choose } \hat{\mathbf{u}} = \mathbf{u}^* \text{ as the coded sequence if} \tag{9}$$
$$\max_{\mathbf{u}} (\sum_{m=0}^{L-1} |\beta_m|)$$
$$= \sum_{m=0}^{L-1} |\int_0^{T_N} \tilde{r}(\tau + T_N)e^{-j\Theta_N(\tau+mT_N, u_m^*)} d\tau|,$$

where $\mathbf{u}^* = (u_0^*, u_1^*, \dots, u_{L-1}^*)$. The decision metric in (10) can also be used for no interleaving case which results in a suboptimum metric.

3 Evaluation of an Upper Bound on the Bit Error Probability

An error event of length L can be described by considering two L-tuples of coded symbols. Let $\mathbf{u}_L = (u_0, u_1, \dots u_{L-1})$ denote transmitting L-tuple and $\hat{\mathbf{u}}_L = (\hat{u}_0, \hat{u}_1, \dots \hat{u}_{L-1})$ denote another L tuple. Each component of $\mathbf{u}_L$ and $\hat{\mathbf{u}}_L$ consists of N coded symbols. An error event with length L occurs when demodulator chooses, instead of transmitted sequence $\mathbf{u}_L$, another sequence $\hat{\mathbf{u}}_L$ of channel symbols corresponding trellis path that splits from the correct path at a given time, and remerges exactly L discrete times later, The union bound provides the following inequality for the bit error probability:

$$P(e) \leq \frac{1}{b}\sum_{L=0}^{\infty}\sum\sum_{\mathbf{u}_L \neq \hat{\mathbf{u}}_L} w(\mathbf{u}_L, \hat{\mathbf{u}}_L)P(\mathbf{u}_L)P(\mathbf{u}_L \to \hat{\mathbf{u}}_L) \tag{10}$$

where $P(\mathbf{u}_L \to \hat{\mathbf{u}}_L)$ is the pairwise error probability and b denotea the number of information bits transmitting every TN *sec.* and $w(\mathbf{u}_L, \hat{\mathbf{u}}_L)$ denotes the hamming distance between two binary sequences corresponding to $\mathbf{u}_L$ and $\hat{\mathbf{u}}_L$.

To find the upper bound on bit error probability in (10), we must first find the pairwise error probability which represent the probability of choosing the coded sequence $\hat{\mathbf{u}}_L = (\hat{u}_0, \hat{u}_1, \dots, \hat{u}_{L-1})$ instead of $\mathbf{u}_L = (u_0, u_1, \dots, u_{L-1})$. Let $|\beta_m|$ denote the maximum likelihood metric for the m + 1st trellis branch of the correct data sequence, computed from (5). Then the pairwise error probability is given by

$$P_r(\mathbf{u}_L \to \hat{\mathbf{u}}_L) = P_r(\sum_{m=0}^{L-1} |\hat{\beta}_m| > \sum_{m=0}^{L-1} |\beta_m||\mathbf{u}_L). \tag{11}$$

Here, $|\hat{\beta}_m|$ denotes the metric computed for the data sequence associated with the m+ 1st trellis branch of the incorrect path. To evaluate (11), we use a linear approximation. Random variable $\hat{\beta}_m$ can be expressed as

$$\hat{\beta}_m = ATu^{(m)} + \tilde{n}_2, \tag{12}$$

where

$$u^{(m)} = \frac{1}{T}\int_0^{T_N} e^{j\Theta(\tau+mT_N, \Delta u_m)} d\tau, \tag{13}$$

and $\Delta u_m = u_m - \hat{u}_m$. We denote the zero mean complex Gaussian random variable, $\tilde{n}_2$, as follows:

$$\tilde{n}_2 = \int_0^{TN} \tilde{n}(\tau + mT_N)e^{-j\Theta_N(\tau+mT_N, \hat{u}_m)} d\tau. \tag{14}$$

For large SNR we can make a linear approximation for $|\hat{\beta}_m|$ as follows:

$$|\hat{\beta}| = |ATu|\sqrt{1 + \frac{u^*\tilde{n}_2 + u\tilde{n}_2^*}{AT|u|^2} + \frac{\tilde{n}_2\tilde{n}_2^*}{|ATu|^2}}$$
$$\approx AT|u|\sqrt{1 + \frac{u^*\tilde{n}_2 + u\tilde{n}_2^*}{AT|u|^2}}$$
$$\approx AT|u| + \frac{1}{2|u|}(u^*\tilde{n}_2 + u\tilde{n}_2^*). \tag{15}$$

Similarly approximation for $|\beta_m|$ can be obtained by setting $\Delta u_m = 0$ which results in $u^{(m)} = N$ and using this in the above expression.

The statistic of $\sum_{m=0}^{L-1}|\hat{\beta}_m| - \sum_{m=0}^{L-1}|\beta_m|$ can be evaluated approximately aa a Gaussian random variable. Define a new random variable Y as

$$Y \triangleq \sum_{m=0}^{L-1}|\hat{\beta}_m| - \sum_{m=0}^{L-1}|\beta_m|. \tag{16}$$

Table 1: Set Partitioning of 2-consecutive Binary CPFSK

level	Partitioning A	Partitioning B	ENSDs	min. ENSD
1	(0, 1) × (0, 1)	(0,1) × (0,1)	1.45,1.63,4.0	1.45
2	0 × (0, 1) 1 × (0,1)	(0, 1) × 0 (0, 1) × 1	1.63	1.63

Then Y can be approximated as a sum of independent Gaussian random variables. We now evaluate the mean, Y, and variance, σ_Y^2, of Y as follows,

$$\bar{Y} = \sum_{m=0}^{L-1} AT(N - |u^{(m)}|), \tag{17}$$

and

$$\sigma_Y^2 = \sum_{m=0}^{L-1} N_0T(N - |u^{(m)}|).$$

Therefore we can rewrite (11) as

$$\begin{aligned} P_r(\mathbf{u}_L \to \hat{\mathbf{u}}_L) &= P_r(Y < 0|\mathbf{u}_L) \qquad (18)\\ &\approx Q\left(\frac{AT\sum_{m=0}^{L-1}(N - |u^{(m)}|)}{\sqrt{N_0T\sum_{m=0}^{L-1}(N - |u^{(m)}|)}}\right) \\ &= Q\left(\sqrt{\frac{\mathcal{E}_s}{N_0}\sum_{m=0}^{L-1}(N - |u^{(m)}|)}\right) \\ &= Q\left(\sqrt{\frac{\mathcal{E}_s}{2N_0}\sum_{m=0}^{L-1} d_{e,m}^2}\right) \end{aligned}$$

where

$$d_{e,m}^2 \triangleq 2(N - |u^{(m)}|). \tag{19}$$

The equivalent normalized squared distance, $d_{e,m}^2$, defined in (19), plays the same role as the normalized squared Euclidean distance of coherent detection for evaluating the error probability of the coded case as well as the uncoded case.

4 Design of Trellis Encoder

It is our goal in this section to design a trellis encoder shown in Figure 1 so that we can get the smallest error probability. As we discussed in the previous section, the pairwise symbol error probability of trellis coded sequences can be expressed as a function of the accumulated ENSD. We define the *equivalent* squared *free distance*, $d_{e,free}^2$, which plays the same role as the squared Euclidean free distance in coherent detection. $d_{e,free}^2$ represents the smallest value of the accumulated ENSDs between sequences. Therefore we should find the encoder having the largest $d_{e,free}^2$. To pursue this goal we use Ungerboeck's set partitioning approach and computer search.

4.1 2-consecutive Binary CPFSK with Modulation Index 1/2

We use the set partitioning method for the set of waveforms of 2-consecutive binary CPFSK. Each waveform is denoted by a two dimensional vector $u_m = (u_{m,0}, u_{m,1})$. As shown in Table 1 we use two level of partitioning. Each subset is denoted by the Cartesian product. The set of vectors in level 1 is partitioned into 2 subsets of size 2.

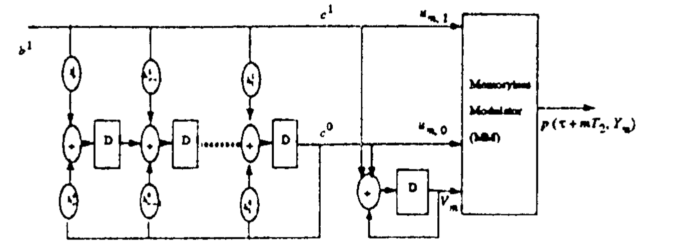

Figure 2: General Representation of Trellis Encoder in Systematic Feedback Form with a Code Rate 1/2, Cascaded to 2CPE of Binary CPFSK

The generation of binary CPFSK is explained in two stages, a *2-consecutive phase encoder 2CPE* and a *memoryless modulator* (MM) as shown in Figure 2. There are two binary input lines in 2CPE. Therefore it is possible to design a binary convolutional encoder with a code rate 1/2, cascaded with 2CPE. We implement this convolutional encoder in systematic feedback form. We write an input sequence, $b^1(D)$, and an output sequence, $c^j(D)$ for j = 0, 1, in polynomial notation. Here D is a delay operator. The information sequence is $c^1(D) = b^1(D)$ and the parity check sequence, $c^0(D)$, is a function of itself and $b^1(D)$. The parity check equation of an encoder describes the relation in time of the encoded bit streams. For an encoder with a code rate 1/2, the parity check equation is

$$H(D)C(D) = O(D) \tag{20}$$

where

$$H(D) = [H_1(D), H_0(D)] \tag{21}$$

is a parity matrix, and

$$C(D) = [c^1(D), c^0(D)]' \tag{22}$$

is an output sequence vector. We define the *constraint length*, ν, to be the maximum degree of all the parity check polynomials $H^j(D)$ for j = 0, 1.

To search for good codes we implement the parity check polynomial in the following form:

$$\begin{aligned} H_1(D) &= h_\nu^1 \cdot D^\nu + h_{\nu-1}^1 D^{\nu-1} + \cdots + h_1^1 + 0, \\ H_0(D) &= h_\nu^0 \cdot D^\nu + h_{\nu-1}^0 D^{\nu-1} + \cdots + h_1^0 + 1. \end{aligned} \tag{23}$$

We assign level 2 subset in Set Partitioning A of Table 1 to the paths leaving the same state of the trellis corresponding to the trellis encoder. This assignment ensures that the ENSD between paths leaving a given state is at least 1.63. This condition is implemented by the connection between the outputs of the systematic convolutional encoder and the inputs of 2CPE as follows: c^0 to $u_{m,0}$ and c^1 to $u_{m,1}$. Therefore the state of the right most memory element of the trellis encoder is decided by c^0 as shown in Figure 2, where $h_\nu^0 = 1$ and $h_0^1 = 0$. The exhaustive search to find the remaining coefficients of $H^j(D)$ for j = 0, 1, to maximize the *equivalent squared free distance*, $d_{e,free}^2$, has been made by means of a computer program.

The results are presented in Table 2 for the number of convolutional encoders with number of states ranging from 2 to 16 states. Only one solution has been reported. For cases where more than one trellis encoder results in maximum $d_{e,free}^2$. If the Set Partitioning B in Table 1 is used, the same results are obtained (with respect to, maximum $d_{e,free}^2$). We have only presented when Set Partitioning A was used.

To evaluate the performance of coded systems, we use two dominant terms in (10) to get an asymptotic upper bound

Table 2: Optimal Code Rate *1/2* Convolutional Encoder Cascaded with 2CP

number of states	v	$[h' \quad h'']$	$d^2_{e,free}$
2	1	[2 1]	3.258
4	2	[2 5]	4.887
8	3	[4 15]	6.341
16	4	(26 21]	7.970

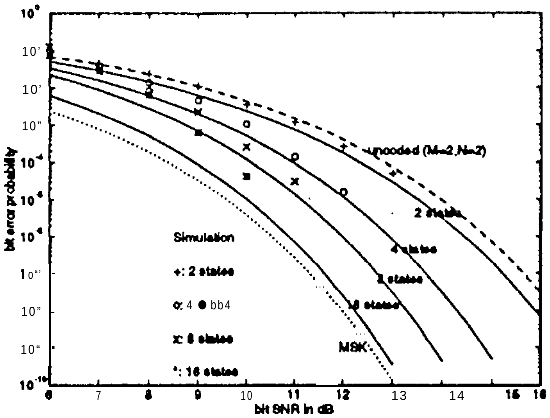

Figure 3: Asymptotic Upper Bound on Bit Error Probability and Simulation Results for 2, 4, 8, and 16 States Trellis Coded Systems of Non-coherent 2-consecutive Binary CPFSK with Code Rate 1/2

on bit error probability. With 2-state trellis encoder in Table 2, we obtain an asymptotic upper bound aa follows:

$$P_b \lesssim Q(0.815\frac{\mathcal{E}_b}{N_0}) + Q(1.178\frac{\mathcal{E}_b}{N_0}). \qquad (24)$$

Observe that $\mathcal{E}_s = 1/2\mathcal{E}_b$ because the code rate is 1/2. With 4-state trellis encoder in Table 2, we obtain an asymptotic upper bound as follows:

$$P_b \lesssim 2Q(1.223\frac{\mathcal{E}_b}{N_0}) + Q(1.541\frac{\mathcal{E}_b}{N_0}). \qquad (25)$$

With 8-state trellis encoder in Table 2, we obtain an asymptotic upper bound as follows:

$$P_b \lesssim 2Q(1.585\frac{\mathcal{E}_b}{N_0}) + 2Q(1.629\frac{\mathcal{E}_b}{N_0}), \qquad (26)$$

and finally 16-state trellis encoder in Table 2, we obtain an asymptotic upper bound as follows:

$$P_b \lesssim 2.5Q(1.993\frac{\mathcal{E}_b}{N_0}) + 0.375\,Q(2.268\frac{\mathcal{E}_b}{N_0}). \qquad (27)$$

Figure 3 shows asymptotic upper bounds and simulation results for 2,4,8 and 16 states trellis coded non-coherent 2-consecutive binary CPFSK. We can observe that simulation result approaches the asymptotic upper bound on bit error probability for large SNR. The dashed line represents the asymptotic upper bound on bit error probability of uncoded non-coherent 2-consecutive CPFSK, by using an l-state r = 1 encoder. By employing trellis coding with a code rate 1/2, we obtain power gains 0.5, 2.25,3.39 and 4.38 dB for 2,4, 8, and 16 states respectively, based on the asymptotic upper bounds at 10^{-5} bit error probability, at a price of reducing the information rate by half. The dotted line represents the asymptotic bit error probability of MSK (coherent binary CPFSK with modulation index 1/2).

Table 3: Set Partitioning of 3-consecutive Binary CPFSK

level	Set partitioning	ENSDs	minimum ENSD
1	(0, 1) x (0, 1) x (0, 1)	1.80, 2.18, 2.76, 4.0, 4.73	1.80
2	0 × (0, 1) × 0, 1 × (0, 1) × 1	2.18, 4.0, 4.73	2.18
	0 × (0, 1) × 1, 1 × (0, 1) × 0	2.18, 2.76, 4.73	
3	0 × (0, 1) × 0	4.73	4.73
	0 × (0, 1) × 1		
	1 × (0, 1) × 0		
	1 × (0, 1) × 1		

4.2 3-consecutive Binary CPFSK with modulation index 1/2

Table 3 shows a set partitioning for the set of waveforms with 3-consecutive binary CPFSK i.e. M = 2 and N = 3. Each signal is denoted by a three dimensional vector $u_m = (u_{m,0}, u_{m,1}, u_{m,2})$, where $u_{m,i} \in$ {0, 1}. We write the set of vectors in the Cartesian product. The set of eight vectors in level 1 is successively partitioned into 2 and 4 subsets of size 4 and 2 respectively.

There are three binary input lines in 3CPE. Therefore it is possible to design a binary convolutional encoder with a code rate 2/3, cascaded with 3CPE. We implement this convolutional encoder in systematic feedback form. We write the input sequences, $b^j(D)$ for j = 1,2, and the output sequences, $c^j(D)$ for j = 0, 1,2 in polynomial notation. Here *D is a* delay operator. The information sequences are $c^j(D)$ = 1?(D) for j = 1,2 and the parity sequence, co (D), is a function of itself and $b^j(D)$ for j = 1,2. The parity check equation of an encoder describes the relation in time of the encoded bit streams. For an encoder with a code rate 2/3, the parity check equation is

$$H(D)C(D) = 0(D) \qquad (28)$$

where

$$H(D) = [H^2(D), H^1(D), H^0(D)] \qquad (29)$$

is a parity matrix, and

$$C(D) = [c^2(D), c^1(D), c^0(D)]' \qquad (30)$$

is an output sequence vector. We define the *constraint length* v to be the maximum degree of all the parity check polynomials $H^j(D)$ for j = 0, 1,2.

To search for good codes we implement the parity check polynomial in the following form:

$$\begin{aligned} H^2(D) &= h^2_v D^v + h^2_{v-1} D^{v-1} + \ldots + h^2_1 + 1, \\ H^1(D) &= h^1_v D^v + h^1_{v-1} D^{v-1} + \cdots + h^1_1 + 0, \\ H^0(D) &= h^0_v D^v + h^0_{v-1} D^{v-1} + \cdots + h^0_1 + 1. \end{aligned} \qquad (31)$$

We assign each subset in level 2 in Table 3 to the paths leaving the same state of the trellis corresponding to the trellis encoder. This assignment ensures that the ENSD between paths leaving a given state is at least 2.18. This condition is implemented by setting h^j_0 = 1 for j = 0,2 and $h^1_0 = 0$. Observe that subsets in level 2 are selected by the value of $u_{m,0} \oplus u_{m,2}$. Therefore the state of the right most memory element of the convolutional encoder is selected by $u_{m,0} \oplus u_{m,2}$ as shown in Figure 4 where $h^1_0 = 0$ and $h^j_0 = 1$ for $j = 0, 2$. The exhaustive search to find the remaining coefficients of $H^j(D)$ for j = 0, 1, 2, which maximize the $d^2_{e,free}$, has been made by means of a computer program. The results are presented in Table 4 for convolutional encoder with the number of statea ranges from 2 to 16 states. Only one solution haa been reported for cases where more than one trellis encoder with maximum $d^2_{e,free}$ was obtained.

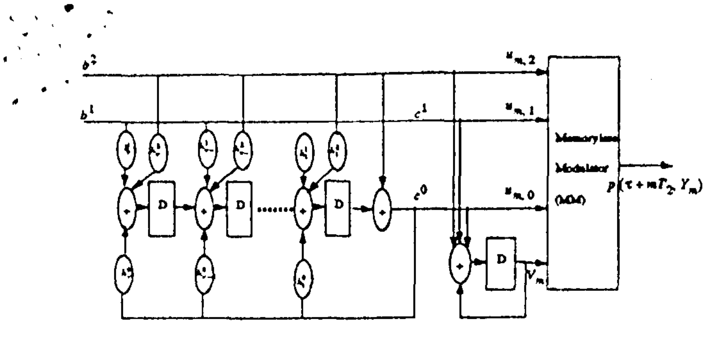

Figure 4: General Representation of Trellis Encoder in Systematic Feedback Form with a Code Rate 2/3, Cascaded with 3CPE of Binary CPFSK

Table 4: Optimal Code Rate 2/3 Convolutional Encoder Cascaded with 3CP

number of states	v	$[h^0\ h^1\ h^2]$	$d^2_{e,free}$
2	1	[1 0 3]	3.983
4	2	[3 6 7]	4.942
8	3	[11 4 9]	6.745
16	4	[19 8 13]	7.965

To evaluate the performance of coded systems we use two dominant terms in (10) to get an upper bound on the bit error probability. With 2-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \tilde{<} 0.75Q(1.328\frac{\mathcal{E}_b}{N_0}) + Q(1.521\frac{\mathcal{E}_b}{N_0}). \quad (32)$$

Observe that $\mathcal{E}_s = 2/3\mathcal{E}_b$ because the code rate is 2/3. With 4-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \tilde{<} 0.75Q(1.647\frac{\mathcal{E}_b}{N_0}) + 0.5\,Q(1.841\frac{\mathcal{E}_b}{N_0}). \quad (33)$$

With 8-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \tilde{<} 0.5Q(2.248\frac{\mathcal{E}_b}{N_0}) + 0.19\,Q(2.374\frac{\mathcal{E}_b}{N_0}). \quad (34)$$

With 16-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \tilde{<} 0.313Q(2.655\frac{\mathcal{E}_b}{N_0}) + 0.875\,Q(2.762\frac{\mathcal{E}_b}{N_0}). \quad (35)$$

Figure 5 shows asymptotic upper bounds and simulation results for 2,4,8 and 16 states trellis coded 2-consecutive binary CPFSK systems. The dashed line represents the asymptotic upper bound on bit error probability of the uncoded noncoherent 3-consecutive CPFSK which is obtained by using $r = 1$, 1-state encoder. By employing trellis coding with a code rate 2/3, we obtain power gains 1.68, 2.63, 3.97 and 4.69 dB for 2, 4, 8, and 16 states respectively based on the asymptotic upper bounds at 10^{-10} bit error probability, but we lose the information rate by 2/3. The dotted line represents the asymptotic bit error probability of MSK (coherent binary CPFSK with modulation index 1/2). Non-coherent coded scheme with 8-state achieves better performance in bit error probability than MSK at an expense of reducing information rate by 2/3.

5 Discussion and Conclusion

We have obtained the coded system with non-coherent detection which has better bit error probability than coherent MSK at an expense of reducing the information rate by a factor equal to the code rate. Furthermore we may achieve a better system than coherent MSK in bandwidth and power efficiency by considering the trellis coding with 4-ary or 8-ary CPFSK with larger number of states.

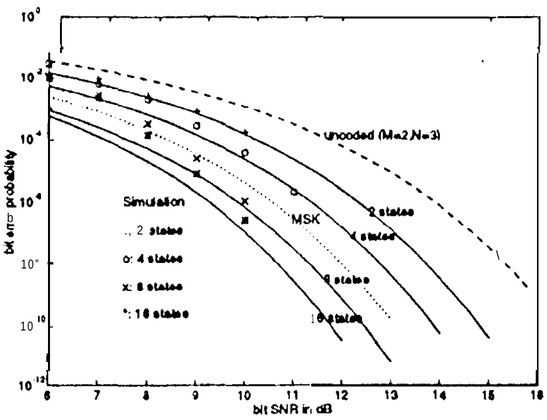

Figure 5: Asymptotic Upper Bound on Bit Error Probability and Simulation Results for 3-consecutive Binary CPFSK Combined with Code Rate 2/3 Convolutional Encoder

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